







Level 3 Calculus, 2009

90638 Manipulate real and complex numbers, and solve equations

Credits: Five 2.00 pm Thursday 26 November 2009

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3-CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	Achievement Criteria				
Achievement	Achievement with Merit	Achievement with Excellence			
Manipulate real and complex numbers, and solve equations.	Solve more complicated equations.	Solve problem(s) involving real or complex numbers.			
Overall Level of Performance					

You are advised to spend 40 minutes answering the questions in this booklet.

QUESTION ONE

(a) Solve $z^2 - 8z + 28 = 0$.

Express your solutions in their simplest form of $z = a \pm b\sqrt{c}i$, where *a*, *b*, *c* are rational numbers.

(b) Solve for *x*:

$$\log_5(8x) - \log_5(3) = \log_5(x+10)$$

(c) Write
$$\left(4\operatorname{cis}\frac{\pi}{6}\right)^5$$
 as a complex number in the form a + bi.

		—
		[

4

(e)) $z = -4 +$	⊦ 5i	is one solution	of the equation
-----	--------------	------	-----------------	-----------------

$$z^3 + Az^2 + 17z - 123 = 0$$

Find the value of A.

<i>u</i> and <i>v</i> are two complex numbers, $v \neq 0$, such that $ u + v ^2 = u ^2 + v ^2$.	L
Prove that $\frac{u}{v}$ is purely imaginary.	
v	

QUESTION TWO

Write $(4 + \sqrt{k})(9 - 2\sqrt{k})$ in the form $a + bk + c\sqrt{k}$, where a, b and c are rational numbers. (a) *u* and *v* are complex numbers where $u = 3Bcis\frac{\pi}{3}$ and $v = Bcis\frac{3\pi}{4}$. (b) Find $u \div v$, expressing your answer in polar form $r \operatorname{cis} \theta$. (c) Solve the following equation for *x*. $\log_4(5-2x) = 3$

Assessor's use only (d) Solve the following equation for x in terms of p.

$$\sqrt{x+p} = \sqrt{x} + 3$$

(e) Solve the following equation for x in terms of k.

$$\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{k}{6}$$

Assessor's use only

Extra paper for continuation of answers if required. Clearly number the question.

